Finite-Time Stability for Non-Gaussian Stochastic Distribution Systems via T-S Fuzzy Modeling

Xiang Gu, Xiaoli Zhang, and Yang Yi
College of Information Engineering, Yangzhou University, Yangzhou, China
Email: 1074841318@qq.com, 1042124805@qq.com, yiyangcontrol@163.com

***Abstract*—In this paper, the problem of finite-time stability for general non-Gaussian stochastic system with unknown state is investigated by using T-S fuzzy modeling. The objective is to control the probability density functions(PDFs) of the system output to follow a given PDFs. In order to describe the gray-box dynamics between PDFs of system output and controlled input, the well known fuzzy logic systems and the T-S fuzzy models are imported at the same time. Then a classical state observer is used to imitate the unknown state of the system. On account of the strong nonlinearity stochastic process, the square root B-spline is used to model the target PDFs. Finally, the favorable finite time stability can be achieved by employing convex optimization theory. Simulations study for a non-gaussian model are given to embody the superiority of the designed algorithm.**

***Index Terms*—T-S fuzzy modeling, finite-time stability, probability density functions(PDFs), unknown state**

I. Introduction

As everyone knows, the study of complex nonlinear stochastic system is of great significance. Therefore in recent years, the relevant research is always a hot spot [1], [2] and many approaches have been applied in this field successfully, such as minimum variance control [3], [4], control for fuzzy stochastic system [5].

In these works, only the output mean and variance are concerned due to the variances which are all obey Gaussian distribution. However, the variances are usually non-gaussian in most practical systems [6]. To solve this problem, a new strategy on the control of the shape of output probability density functions(PDFs) for general stochastic system have been developed [7].This kind of control method in which the whole shape of the output PDF is considered is called stochastic distribution control (SDC) or PDF tracking control [8]. Due to the probability density function is nonlinear and must be satisfy the restriction condition, it is difficult to get the exact expression of probability density function, so the probability density function can only be modeled by approximate model, a suitable approximation method becomes particularly important. Currently, the most commonly used approximation method is B-spline expansion [9], [10].

All along fuzzy control systems have been believed to be effective tools when dealing complex controlled processes even some nonanalytic system. [11]

As two most typical examples in various fuzzy models, T-S fuzzy models and fuzzy logic systems (FLSs) have received a good deal of attention in different research fields [12]-[15]. In fact, an FLS can effectively approximate any continuous functions between control inputs and system outputs [16]. A series of fuzzy rules acquired from human experts are correlated with fuzzy basic functions by using FLSs. As for famous T-S fuzzy models, the local nonlinear dynamics can be modeled by a set of simple linear models in diverse spaces and by combing membership functions with those involved local models, the global fuzzy models can be effective for many control systems or even describe those nonlinear dynamics exactly [17]-[19].

On the other hand, in order to meet the practical requirements of system disturbance rejection in industrial production, the stability analysis of a system should not be limited to the stability ability of infinite time, but should consider its stability within the required time. Therefore, issues of finite-time control have became more and more popular [20], [21]. Ref. [22] addresses the challenging problem of finite-time fault tolerant attitude stabilization control for the rigid spacecraft attitude control system with external disturbances and actuator failures. In Ref. [23], the finite-time containment control for a second-order nonlinear multi-agent system in the presence of external disturbances was discussed. In addition, the state observer [24] is built to deal with the unknown system state.

Motivated by the above observations, a method based on T-S fuzzy modeling and state observer method is proposed for a non-gaussian stochastic distribution system with unknown state. First, the square root B-spline is used to identify the nonlinear dynamics from the dynamical weights to the controlled input. Then the

Manuscript received December 27, 2019; revised May 12, 2020.

doi: 10.18178/joace.8.1.9-14

two-step fuzzy modeling is designed to achieve the system global fuzzy model. With the help of the state observers, the error equation between state estimation and actual system is established. By applying the state feedback control law and the convex optimization algorithm, the favorable finite-time stability can be achieved in the limited time domain, which greatly improves the realistic reliability of the system. Finally, simulations result for aforesaid model are provided to reflect the validity of designed algorithm.

Nomenclature. If not stated, all involved vectors or matrices are supposed to have suited dimensions. The identity and zero matrices are respectively represented by ***I*** and ***0***. For any matrices P, the mark sym is defined as $sym(P) = P + P^T$.

II. System Description

For a dynamic stochastic system, denote $u(t) \subset R^m$ as the control input, $\eta(t) \subset [a,b]$, as the stochastic output and the probability of output $\eta(t)$ lying inside $[a,\sigma]$ can be described as

$$P(a \le \eta(t) < \sigma, u(t), d(t)) = \int_a^\sigma \gamma(y, u(t), d(t))dy \tag{1}$$

where $\gamma(y,u(t),d(t))$ is the output PDF of the stochastic variable $\eta(t)$ under control input $u(t)$ and disturbance $d(t)$. As in [4], [5], it is supposed that the output PDF $\gamma(y,u(t),d(t))$, as the control objective, can be measured or estimated. For PDF $\gamma(y,u(t),d(t))$,the square root B-spline expansion is given by

$$\sqrt{\gamma(y,u(t),d(t))} = \sum_{i=1}^{n} v_i(u(t))B_i(y) \tag{2}$$

where $B_i(y)(i=1,2,...,n)$ are specified basis functions and $v_i := v_i(u(t))(i=1,2...,n)$ are the corresponding weights which depend on $u(t)$. It can be seen that the positiveness of $\gamma(y,u(t),d(t))$ can be automatically guaranteed. On the other hand, the PDF should satisfy the condition $\int_a^b \gamma(y,u(t),d(t))dy = 1$ which means only n-1 weights are independent. So the square root expansions are considered as follows

$$\gamma(y,u(t),d(t)) = (C_0V(t) + L(y))^2 \tag{3}$$

where $C_0 = [C_1(y)C_2(y)...C_{n-1}(y)]$, and $V(t) = [v_1(t)v_2(t)...v_{n-1}(t)]^T$.

The above formula reflects the relationship between the state variables of the weight coefficient system and the control input, and also describes the approximation of the b-spline function to the probability density function, and it can be seen that once a set of basis functions is selected, then *C0(y)* and *L(y)* will be determined.

Then the typical T-S models will be used to represent the unmodeling dynamics and the ith fuzzy rule of T-S weight models is expressed as

Plant Rule i : If ϑ_1 is k_{i1}, ϑ_2 is k_{i2} and ... and ϑ_q is k_{iq}, then:

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + C_i d(t) \\ V(t) &= E_i x(t) \end{aligned} \tag{4}$$

where $x(t)$ is the unknown middle state, $u(t) \in \mathbf{R}^m$ is the controlled input. $V(t) \in R^{n-1}$ are the independent weight vectors, $d(t)$ represent the unknown exogenous disturbance. A_i, B_i, C_i and E_i are system matrices with appropriate dimension. k_{ij} , ϑ_j are the priori variables and the fuzzy sets respectively. p is the number of the rules. q stands for the number of the priori variables. Similarly with AAA, and by using fuzzy blend, the global weight models can be expressed as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{p} h_i(\vartheta)(A_i x(t) + B_i u(t) + C_i d(t)) \\ V(t) &= \sum_{i=1}^{p} h_i(\vartheta) E_i x(t) \end{aligned} \tag{5}$$

Where $\vartheta = [\vartheta_1, ...\vartheta_q]$, $h_i(\vartheta) = \sigma(\vartheta) / \sum_{i=1}^{p} \sigma_i(\vartheta)$, $\sigma_i(\vartheta) = \prod_{j=1}^{q} k_{ij}(\vartheta_j)$ and $k_{ij}(\cdot)$ are the grades of the function of k_{ij} . Furthermore, $h_i(\vartheta)$ can satisfy the following condition

$$h_i(\vartheta) \ge 0, i = 1...p. \sum_{i=1}^{p} h_i(\vartheta) = 1 \tag{6}$$

It is obvious that the existence of unknown disturbances seriously affects the dynamical performance of the system. Furthermore, in many real control systems, the systems state always unknown, which causes a lot of difficulties in controller design and disturbance rejection.

III. Controller and Observer Design Method

It is noted that the middle state $x(t)$ in weight models is unmeasurable and only the weight $V(t)$ and the output PDFs can be employed. For evaluate the unknown state in the controlled process, the available state observer is designed as

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{p} h_i(\vartheta)(A_i\hat{x}(t) + B_iu(t) + \\ &L(\sqrt{\gamma(y,u(t),d(t))} - \sqrt{\hat{\gamma}(y,u(t))}) \\ &= \sum_{i=1}^{p} h_i(\vartheta)(A_i\hat{x}(t) + B_iu(t) + \\ &L(C_0\sum_{i=1}^{p} h_i(\vartheta)E_i(x(t) - \hat{x}(t)) \\ V(t) &= \sum_{i=1}^{p} h_i(\vartheta)E_i\hat{x}(t)\end{aligned} \tag{7}$$

where $\hat{x}$ stands for the estimation of the system state. By defining $e(t) = x(t) - \hat{x}(t)$ and $\bar{E} = \sum_{i=1}^{p} h_i(\vartheta)E_i$. we can obtain

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= \sum_{i=1}^{p} h_i(\vartheta)(A_i\hat{x}(t) + B_iu(t) + C_id(t)) \\ &- \sum_{i=1}^{p} h_i(\vartheta)(A_i\hat{x}(t) + B_iu(t) \\ &+ L(\sqrt{\gamma(y,u(t),d(t))} - \sqrt{\hat{\gamma}(y,u(t))}) \\ &= \sum_{i=1}^{p} h_i(\vartheta)(A_ie(t) + C_id(t) - LC_0\bar{E}e(t)\end{aligned} \tag{8}$$

In order to realize the satisfactory control effect. The controller is constructed as

$$u(t) = -K_i\hat{x} \tag{9}$$

where K_i are the controller gains to be computed later. By using fuzzy rules, the overall controller is expressed as

$$u(t) = \sum_{i=1}^{p} h_i(\vartheta)(-K_i\hat{x}) \tag{10}$$

Substituting the overall fuzzy controller (10) into the weights models (5), the corresponding closed-loop system is given by

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{p} h_i(\vartheta)h_j(\vartheta)(A_ix(t) - B_iK_i\hat{x} + C_id(t)) \\ &= \sum_{i=1}^{p} h_i(\vartheta)h_j(\vartheta)((A_i - B_iK_j)x(t) \\ &- B_iK_ie(t) + C_id(t))\end{aligned}$$

$$V(t) = \sum_{i=1}^{p} h_i(\vartheta)E_ix(t) \tag{11}$$

IV. Main Results

In order to design effective control input, the following assumption needs to be given.

Assumption : The exogenous disturbance $d(t)$ is assumed as satisfy $\int_0^T d^T(s)d(s)ds < \tau$ where τ is a positive scalar.

Definition: For a given normal number $c_1, c_2(> c_1), \rho, T$ and a positive definite symmetric matrices R_0, the closed-loop system is finite-time stable if this is true for $x^T(0)R_0x(0) \le c_1$, then for all $t \in [0,T]$, we have $x^TR_0x \le c_2$.

Theorem: For the designed parameters $\lambda > 0$, $\sigma > 0$ if we can find matrices $Q = P_1^{-1} > 0$ and R_1, R_2 such that the following inequalities are solvable, and $\Psi_1 = h_i(\vartheta)h_j(\vartheta)sym(P_1(A_i - B_iK_i)) - \sigma P_1$, $i = 1,2...p,\ j = 1,2...p$

$$\begin{bmatrix}\Psi_1 & h_i(\vartheta)h_j(\vartheta)P_1B_iK_j & P_1C_i \\ * & \Psi_2 & P_2C_i \\ * & * & -\sigma I\end{bmatrix} < 0 \tag{12}$$

$$c_1\lambda_1 + \tau(1 - e^{-\sigma T}) \le \lambda_2c_2e^{-\sigma T}$$

$\bar{P} = R_0^{-1/2}PR_0^{-1/2}$, $P = diag[P_1, P_2]$, $\Psi_2 = h_i(\vartheta)sym(P_2(A_i - LC_0\bar{E}) - \sigma P_2, i = 1,2..p$, $\lambda_1 = \lambda_{max}(\bar{P})$, $\lambda_2 = \lambda_{min}(\bar{P})$ then the tracks of the state of system (4) is finite-time stable. The gain matrix is given by $K = R_1Q^{-1}$, $L = P_2^{-1}R_2$.

Proof: Design a suited Lyapunov function as

$$\begin{aligned}\Phi_1(x(t)) &= x^T(t)P_1x(t) \\ \Phi_2(e(t)) &= e^T(t)P_2e(t)\end{aligned} \tag{13}$$

The derivative of Φ_1 and Φ_2 is deduced as

$$\begin{aligned}\dot{\Phi}_1 &= \sum_{i,j=1}^{p} h_i(\vartheta)h_j(\vartheta)(x^Tsym(P_1(A_i - B_iK_i))x \\ &+ 2x^TP_1B_iK_je) + 2\sum_{i=1}^{p} h_i(\vartheta)x^TPC_id(t) \\ \dot{\Phi}_2 &= \sum_{i=1}^{p} h_i(\vartheta)e^Tsym(P_2(A_i - LC_0\bar{E}))e \\ &+ 2\sum_{i=1}^{p} h_i(\vartheta)e^TP_2C_id(t)\end{aligned} \tag{14}$$

select Lyapunov functions as

$$\Phi_3 = \Phi_1 + \Phi_2 \tag{15}$$

Then combing (14)

$$\dot{\Phi}_3 \le \rho^T(t)\begin{bmatrix} \Psi'_1 & \sum_{i,j=1}^{p} h_i(\vartheta)h_j(\vartheta)P_1B_iK_j \\ * & \Psi'_2 \end{bmatrix}\rho(t) + 2\rho^T PBd(t) \tag{16}$$

Where

$$\Psi'_1 = \sum_{i,j=1}^{p} h_i(\vartheta)h_j(\vartheta)sym(P_1(A_i - B_iK_i))$$

$$\Psi'_2 = \sum_{i=1}^{p} h_i(\vartheta)sym(P_2(A_i - LC_0\bar{E}) \tag{17}$$

Then we construct the following formula

$$\dot{\Phi}_3 \le \sigma\Phi_3 + \sigma d^T(t)d(t) \tag{18}$$

Multiply both sides of (18) by $e^{-\sigma t}$

$$\frac{d}{dt}(e^{-\sigma t}\Phi_3(t)) \le \sigma e^{-\sigma t}d^T(t)d(t) \tag{19}$$

Integrate both sides of (19)

$$\Phi_3(t) \le e^{\sigma T}\Phi_3(0) + \sigma e^{-\sigma T}\int_0^t e^{-\sigma T}d^T(s)d(s))ds \le e^{\sigma T}\lambda_1 c_1 + e^{\sigma T}\tau(1 - e^{-\sigma T}) \tag{20}$$

On the other hand

$$\Phi_3(t) \ge \lambda_2\rho^T(t)R_0\rho(t) \tag{21}$$

Combing (20) and (21), we can get

$$\rho^T(t)R_0\rho(t) \le \frac{\lambda_1 c_1 + \tau(1 - e^{-\sigma T})}{\lambda_2 e^{-\sigma T}} \tag{22}$$

Then by using (12) we can get $\rho^T(t)R_0\rho(t) \le c_2$.

V. Numerical Illustrations

In this section, a stochastic system with non-Gaussian process is considered. It is supposed that the output PDF can be formulated to be (3) with $B_i = sin2\pi y, y \in [0.5(i-1), 0.5i], i = 1,2,3$ otherwise $B_i = 0$, besides $C_0(y) = [B_1(y) - B_4(y)\ \ B_2(y) - B_4(y)\ \ B_3(y) - B_4(y)]$, $L(y) = B_4(y) / \int_0^{1.5} B_4(y)$. The dynamical relations between V and u is describe as

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 0.6 & -2 & -0.8 \\ 0.5 & 0.4 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0.5 & o & o1 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & 0.5 & 1 \\ 0.5 & -1 & 0.5 \\ 0 & 0.7 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

In order to highlight the universality, our motivation is to choose most representative or common functions as fuzzy membership functions in the process of identification. The two Gaussian-type basis functions are considered as the following fuzzy sets $\kappa_{i1} = exp(-(\omega_1 - 1)^2\varepsilon^{-2}), \kappa_{i2} = exp(-(\omega_1 + 1)^2\varepsilon^2)$ where $\varepsilon = 0.8$. So the member functions can be computed as

$$h_1(\omega_1) = \frac{exp(-(\omega_1 - 1)^2\varepsilon^2)}{exp(-(\omega_1 - 1)^2\varepsilon^{-2}) + exp(-(\omega_1 + 1)^2\varepsilon^2)}$$

$$h_2(\omega_1) = \frac{exp(-(\omega_1 + 1)^2\varepsilon^2)}{exp(-(\omega_1 - 1)^2\varepsilon^{-2}) + exp(-(\omega_1 + 1)^2\varepsilon^2)}$$

Obviously, $h_1(\omega_1) + h_2(\omega_1) = 1$.Then computing (12), the gain matrices K and L are found to be

$$K_1 = \begin{bmatrix} -20.2876 & 3.1993 & -16.00579 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -25.3131 & -10.0489.1993 & -18.0293 \end{bmatrix}$$
$$L = \begin{bmatrix} 0.0049 & -0.003 & 0.0043 \end{bmatrix}$$

Supposed that the initial values $x(0) = \hat{x}(0) = [0.1:0.1:0.2]$. When the controller is applied, the state trajectory of the stochastic system is shown in Fig. 1. Fig. 2 displays the satisfactory estimation for unknown state, which means that the proposed state observer turns out to be effective. The practical PDFs for the weighting system and under the proposed robust control strategy is shown in Fig. 3.

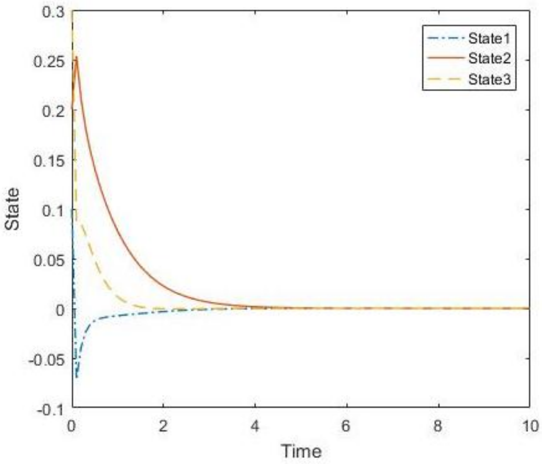

Figure 1. The state of the system.

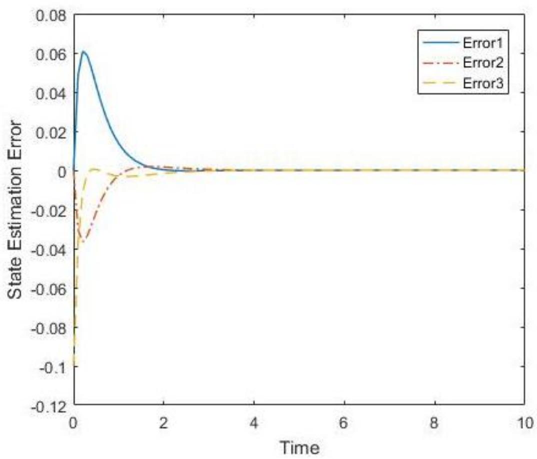

Figure 2. The state estimation error.

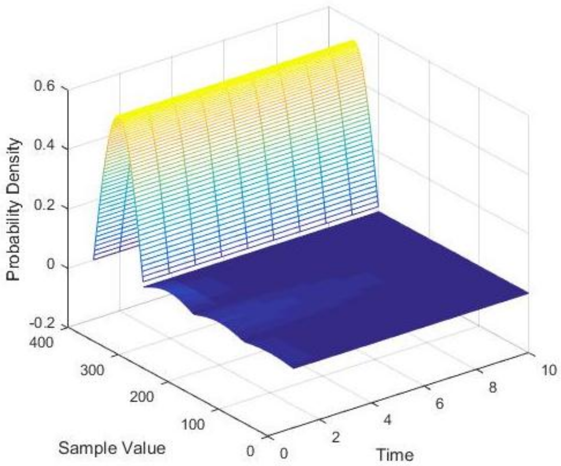

Figure 3. 3-D plot of the output PDFs.

VI. Conclusion

This paper uses the T-S fuzzy model to study the finite-time stability of general non-Gaussian stochastic systems with unknown states. The purpose is to control the output PDFs to follow a given PDF, and the well-known fuzzy logic system and T-S fuzzy model were introduced to model the system output PDF and the controlled input. Then, use the classic state observer to mimic the unknown state of the system. Finally, by using convex optimization theory, the favorable finite-time stability can be achieved. The simulation study of the non-Gaussian model shows the superiority of the designed algorithm.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Gu xiang complete the design and compilation of the paper, Zhang xiao li set up and debug the simulation. Yi Yang , as the teacher, supervise and correct the paper.

Acknowledgment

This work was supported in part by the National Nature Science Foundation of China under Grants (61803331,61973266),the Nature Science Foundation of Jiangsu Province under Grant (BK20170515) and the QingLan Project of Jiangsu Province.

Reference

[1] J. H. Wang, Z. L and Y. Z, "Extended dimension fuzzy adaptive control for nonlinear uncertain stochastic systems with actuator constraints," *Nonlinear Dynamics*, vol. 98, pp. 1315-1329, Oct, 2019.

[2] Z. Namadchian and M. Rouhani, "Observer-based adaptive neural control for switched stochastic pure-feedback systems with input saturation," *Neurocomputing*, vol. 375, 80-90, Jan 2019.

[3] Brian, "Introduction to stochastic control theory," *IEEE Transactions. on Automatic Control*, vol. 17, 422- 423, July. 1972

[4] K. J. Astrom and B. Wittenmark, "Adaptive control," *Reading* 1989.

[5] Z. Wang, D. W. C. Ho and X. Liu, "A note on the robust stability of uncertain stochastic fuzzy systems with time-delays," *IEEE Transactions on Systems Man and Cybernetics - Part A Systems and Humans*, vol. 34, pp. 570-576, Aug, 2004.

[6] Y. Yi, H. Shen, and L. Guo, "Constrained PID tracking control for output PDFS of non-gaussian stochastic system based on LMIs," *Asian J. Control*, vol 5, 571–577, Nov, 2009.

[7] O. Elbeyli, L. Hong and J. Q. Sun, "On the feedback control of stochastic systems tracking pre-specified probability density functions," Inst.Meas.Control, vol 27, 319-329, Feb, 2005.

[8] Y. Wu and J. X. Dong, "Fault detection for non-gaussian stochastic distribution fuzzy systems by an event-triggered mechanism," *ISA Transactions*, vol 91, 135-150, Aug, 2019

[9] L. Guo and H. Wang, "Generalized discrete time PI control of output PDFs using square root b-spline expansion," *Automatica*, vol. 41, 159-162, Jan. 2005.

[10] L. Guo and H. Wang, "Fault detection and diagnosis for general stochastic systems using B-spline expansions and nonlinear filters," in *Proc. of the IEEE Conference on Decision and Control*, Seville, 2005, 4782- 4787.

[11] G. Feng, "Analysis and synthesis of fuzzy control systems: model based approach," Boca Raton: CRC, 2010, ch. 2.

[12] S. C. Tong, Y. M. Li and S. Sui, "Adaptive fuzzy output feedback control for switched non-strict-feedback nonlinear systems with input nonlinearities," *IEEE Trans. Fuzzy Syst*, vol 24, 1426-1440, Dec, 2016.

[13] Y. M. Li and S. C. Tong, "Adaptive fuzzy output-feedback stabilization control for class of switched nonstrict-feedback nonlinear systems," *IEEE Trans. Cybern*, vol 47, 1007-1016, Apr, 2017.

[14] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst Man Cybern*, vol. 15, pp. 116-132, Jan. 1985.
[15] N. K. Kwon, I. S. Park, and P. K. Park, "H∞ state-feedback control for continuous-time Markovian jump fuzzy systems using a fuzzy weighting-dependent Lyapunov function," *Nonlinear Dyn*, vol. 90, pp. 2001-2011, Nov, 2017.
[16] N. R. Cazarez-Castro, L. T. Aguilar, and O. Castillo, O, "Designing type-1 and type-2 fuzzy logic controllers via fuzzy Lyapunov synteesis for non-smooth mechanical systems," *Egn. Appl. Artif. Intell*, vol. 25, pp. 971-979, Aug. 2012.
[17] Y. Yi, X. X. Fan, and T. P. Zhang, "Anti-disturbance tracking control for systems with nonlinear disturbances using T-S fuzzy modeling," *Neurocomputing*, vol. 171, pp. 1027-1037, Jan. 2016.
[18] Y. Yi, W. X. Zheng, C. Y. Sun, and L. Guo, "DOB fuzzy controller design for non-Gaussion stochastic distribution systems using two-step fuzzy identification," *IEEE Trans. Fuzzy Syst*, vol 24, 401-418, Jul, 2015.
[19] Y. Yi, L. R. Shao, and X. X. Fan, "Tolerant control for non-Gaussian stochastic processes with unknown faults via two-step fuzzy modeling," *Nonlinear Dyn*, vol. 95, pp. 1027-1037, Dec, 2018.
[20] X. Liu, N. Jiang, and J. Cao, "Finite-time stochastic stabilization for BAM neural networks with uncertainty," *Journal of the Frabklin Institute*, vol. 350, pp. 2109-2123, Oct. 2013.
[21] Y. H. Liu, H. Li, and Q. S. Zhong, "Robust finite-time H∞ control for nonlinear system with parameter uncertainties and one-side Lipschitz via output feedback," in *Proc. 36th China Control Conference*, Dalian, China, 2017, 433-438.
[22] B. Li, K. Qin, B. Xiao, and Y. S. Yang, "Finite-time extended state observer based fault tolerant output feedback control for attitude stabilization," ISA transactions, vol. 91, pp. 11-20, Aug, 2019.
[23] H. Lu, W. L. He, and Q. L. Han, "Finite-time containment control for nonlinear multi-agent systems with external disturbances," *Information Sciences*, vol. 2020, pp. 338-351, Feb, 2020.
[24] R. M. Brisilla and V. Sankaranarayanan, "Extended state observer-based sliding mode control for multi-input multi-output system with multiple disturbances," *Journal of Control, Automation and Electrical Systems*, vol. 28, pp. 11-25 Nov. 2017.

Xiang Gu received the B.Sc. degree from the College of electrical engineering, Changzhou Institute Of Technology, changzhou, China, in 2017. Now, he is a postgraduate student in Yangzhou University. His current research interests include robust control and anti-disturbance control.

Yang Yi received the M.Sc degree from the School of Information Engineering, Yangzhou University, Yangzhou, China, in 2005, and Ph.D. degree from the School of Automation, Southeast University, Nanjing, China, in 2009. Now, he is an associate professor in Yangzhou University. From July 2012 to July 2013, he was a visiting scientist in the School of Computing, Engineering and Mathematics, University of Western Sydney, Australia. His current research interests include stochastic systems, intelligent systems and anti-disturbance control.